Mode Coupling by a Longitudinal Slot for a Class of Planar Waveguiding Structures: Part I—Theory

PERRY F. WILSON, MEMBER, IEEE, AND DAVID C. CHANG, FELLOW, IEEE

Abstract—**Coupling between two parallel-plate waveguides is investigated. Mutual excitation is due to a longitudinal slot in a common plate. The introduction of reflecting boundaries parallel to the slot allows one to model a number of planar waveguiding structures featuring a common coupling mechanism. Part I of this paper details the analysis of the basic slot scattering problem based on the singular integral equation method. If one assumes that the slot is small, then closed-form algebraic modal equations follow. These modal equations are well-adapted to numerical parametric studies.**

I. Introduction

LONGITUDINAL SLOTS can be used to transfer energy between waveguides (directional couplers) and to control modal properties (e.g., the ridge guide, finline, etc.). Unlike electric current probes, slots or the equivalent magnetic current "probes" can be fabricated in the planar direction. Hence, slots are consistent with integrated-circuit technology for microwave and millimeter-wave applications. An exact analysis of even simple geometries can prove formidable, however. In treating directional couplers, electrically short slots are typically treated as radiators, either via the small aperture (obstacle) theory pioneered by Lamb [1], Rayleigh [2], [3], and Bethe [4], [5], or by employing some dynamic modification to treat medium to resonant length slots [6]–[13]. For slots wavelengths long, the fields in the two waveguide regions may be described in terms of the interference between a pair of propagating system modes for the composite structure [11], [14].

The present paper analyzes modal properties for a class of slotted waveguide structures coupled together via a narrow longitudinal slot. Rather than consider any specific waveguide geometry directly, we first investigate the slot excitation problem in isolation. Specifically, we examine the manner in which a pair of slot-coupled, dielectric-loaded, parallel-plate waveguides scatters an obliquely incident TEM mode. Introducing reflecting boundaries and picturing the TEM mode as being bounced back and forth, as well as undergoing a slot scattering, leads to a transverse resonance description of a larger system's modal properties. This approach is often mentioned as a method of investigating modes in rectangular waveguides (cf. [15]), dielectric slab guides [16], and more recently, microstrips [17]–[19], and open dielectric waveguides [20], [21].

The types of planar structures which can be treated by this method include, as shown in Fig. 1, the rectangular coaxial transmission line (shielded stripline), broadwall coupled rectangular waveguides (shielded slotline), coupled microstrips, and coupled microstrip and rectangular waveguide. Each features a slot (slots) between planar sections,

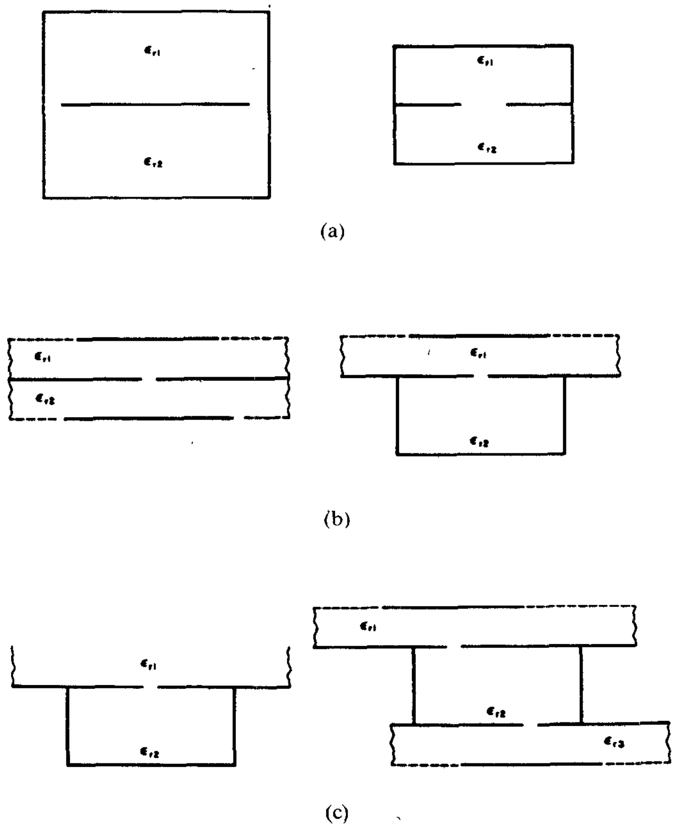


Fig. 1. Planar waveguiding structures featuring longitudinal slot coupling. (a) Rectangular coaxial transmission line. (b) Coupled microstrips. (c) Microstrip feeding an infinite half-space. (d) Shielded slotline. (e) Coupled rectangular waveguide and microstrip. (f) Microstrips about a rectangular waveguide.

Manuscript received February 28, 1985, revised June 5, 1985. This work, performed at the University of Colorado, was supported in part by the National Bureau of Standards and monitored by Drs. M. T. Ma and M. Kanda.

P. F. Wilson is with the Electromagnetic Fields Division, National Bureau of Standards, Boulder, CO 80303.

D. C. Chang is with the Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309.

0018-9480/85/1000-0981$01.00 ©1985 IEEE

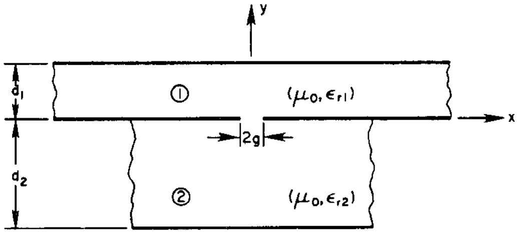


Fig. 2. The slot scattering cross section.

possibly loaded, with reflecting boundaries parallel to, and away from, the slot. Additional structures are possible, as will be discussed in Part II of the paper.

The method of solution is to formulate a set of integral equations for the fields excited in the slot. The kernels (singular) are simplified by requiring that the slot be "narrow," that is, electrically narrow and small compared to the primary waveguide dimensions. This yields a set of approximate integral equations which may be solved in closed form. The slot excitation is then characterized in terms of a scattering matrix. A specific waveguide is modeled by introducing the appropriate reflection matrix. Imposing the transverse resonance condition then determines the proper modal equation. The roots of these modal equations must be found numerically. Fortunately, they are relatively simple. Thus, the present approach is well-suited to problems where numerous parameter changes are of interest. A complete modal description is not attempted here. Rather, we restrict our attention to LSM-type modes with resonances only in the transverse direction. Typically, this description covers the dominant TE, or quasi-TEM, mode. A more general treatment is possible, but at the expense of the simple, closed-form solution developed here. Part I of this two-part paper presents the formal derivation leading to the modal equation. Actual applications to waveguides like those depicted in Fig. 1 are the subject of Part II.

II. Integral Equation Formulation

The cross section to be analyzed is depicted in Fig. 2. The upper and lower regions will be denoted by either a subscript or superscript j, with $j=1$ referring to the upper guide, and $j=2$ the lower. Each parallel-plate region is allowed to have an arbitrary dielectric constant ϵ_{rj} and parallel-plate separation d_j. The slot width is $2g$ with the $x-y$ coordinates as shown. All field quantities are assumed to propagate according to $\exp i(k_0\alpha z-\omega t)$, where the z-axis is along the slot center.

The incident TEM mode in region j strikes the slot at an angle θ_j, where θ_j is measured from the x-axis. The normalized propagation constant α is therefore related to θ_j via $\alpha = n_j \sin\theta_j$, where n_j is the refractive index. Introducing the Fourier transform pair

$$f(x,y)=\int_{-\infty}^{\infty}\tilde{f}(\beta,y)e^{ik_0\beta x}\,d\beta = F[\tilde{f}]$$
$$\tilde{f}(\beta,y)=\frac{k_0}{2\pi}\int_{-\infty}^{\infty}f(x,y)e^{-ik_0\beta x}\,dx = F^{-1}[f] \quad (1)$$

removes the x-variation.

Because of the dielectric interface, it is natural to decompose the fields into longitudinal section modes. In general, both LSE and LSM modes will be present; however, because the incident fields are of the LSM type ($Hy=0$), the LSE fields will be significant only in the slot region. Solving the appropriate wave equation in the transform domain yields the following expressions for the primary field components of interest here [27]:

$$(\pm)\tilde{\boldsymbol{H}}_t^{(j)}(\beta,y)=i\tilde{\phi}^{(j)}(\beta,y)\left\{\frac{\epsilon_{rj}}{\eta_0}\tilde{\boldsymbol{E}}_t^{(j)}(\beta,0)\times \boldsymbol{a}_y - (\alpha\boldsymbol{a}_z+\beta\boldsymbol{a}_x)\tilde{H}_y^{(j)}(\beta,0)\right\}$$
$$(\pm)\tilde{E}_y^{(j)}(\beta,y)=i\tilde{\phi}^{(j)}(\beta,y)\cdot\left\{(\alpha\boldsymbol{a}_z+\beta\boldsymbol{a}_x)\cdot\tilde{\boldsymbol{E}}_t^{(j)}(\beta,0)\right\} \quad (2)$$

where

$$\tilde{\phi}^{(j)}(\beta,y)=\frac{\cosh k_0u_j(d_j\mp y)}{u_j\sinh k_0u_jd_j} \quad (3)$$

and $u_j=(\alpha^2+\beta^2-n_j^2)^{1/2}$. The subscript t denotes the tangential $x-z$ plane. The signs in parenthesis refer to the upper ($j=1$) and lower ($j=2$) regions. Later, an additional sign pair will be introduced associated with x-directed forward or backward propagation. These signs will not be in parenthesis and the distinction should remain apparent. It should be noted that $\tilde{H}_y^{(j)}$ and $\tilde{\boldsymbol{E}}_t^{(j)}$ could be similarly expressed; however, these components are of secondary interest here.

Before matching the fields across the slot, we may exit the transform domain by employing the following relationship:

$$F[(\alpha\boldsymbol{a}_z+\beta\boldsymbol{a}_x)\tilde{f}]=\frac{-i}{k_0}\nabla_t F[\tilde{f}]. \quad (4)$$

This result, together with a convolution theorem, i.e., $F[\tilde{f}\tilde{g}]=f*g$, allows the above scattered fields to be written

$$(\pm)\boldsymbol{H}_t^{(j)}(x,y)=i\int_{-g}^{g}\left\{\frac{\epsilon_{rj}}{\eta_0}\phi^{(j)}(x-x',y)\boldsymbol{E}_t^{(j)}(x',0)\times\boldsymbol{a}_y + \frac{i}{k_0}\nabla_t\phi^{(j)}(x-x',y)H_y^{(j)}(x',0)\right\}dx'$$
$$(\pm)E_y^{(j)}(x,y)=\frac{1}{k_0}\nabla_t\int_{-g}^{g}\left\{\phi^{(j)}(x-x',y)\boldsymbol{E}_t^{(j)}(x',0)\right\}dx' \quad (5)$$

where we have noted that both $\boldsymbol{E}_t^{(j)}(x',0)$ and $H_y^{(j)}(x',0)$ are zero outside the slot. To avoid any ambiguity concerning integrability, the tangential del operators should be applied after the x'-integration and is included inside the brackets in the $\boldsymbol{H}_t^{(j)}(x,y)$ expression only for notational simplicity.

We next wish to impose continuity across the slot. Normally, it is sufficient to match the tangential fields. However, as indicated by Bethe [4] and others [22]–[24], an additional continuity condition on the normal components may be necessary to develop a consistent small aperture solution. The continuity of D_y, E_t, and $\boldsymbol{H}$ yields the following set of integral equations:

$$i\int_{-g}^{g}\left\{\frac{1}{\eta_0}\left[\epsilon_{r1}\phi^{(1)}(x-x')+\epsilon_{r2}\phi^{(2)}(x-x')\right]\boldsymbol{E}_t(x')\times\boldsymbol{a}_y + \frac{i}{k_0}\nabla_t\left[\phi^{(1)}(x-x')+\phi^{(2)}(x-x')\right]H_y(x')\right\}dx' = \boldsymbol{H}_{t,\mathrm{inc}}^{(2)}(x)-\boldsymbol{H}_{t,\mathrm{inc}}^{(1)}(x)$$

$$\frac{1}{k_0}\nabla_t\int_{-g}^{g}\left\{\left[\epsilon_{r1}\phi^{(1)}(x-x')+\epsilon_{r2}\phi^{(2)}(x-x')\right]\cdot\boldsymbol{E}_t(x')\right\}dx' = \epsilon_{r2}E_{y,\mathrm{inc}}^{(2)}(x)-\epsilon_{r1}E_{y,\mathrm{inc}}^{(1)}(x) \quad (6)$$

where all quantities are understood to be in the slot ($y=0$) and "inc" denotes the incident fields.

The kernel $\phi^{(J)}(x,y)$ may be evaluated via a residue calculation; deforming the path of integration into the upper (+forward) or lower (−backward) half-plane yields

$$\phi^{(J)\pm}(x,y)=\frac{i\pi}{k_0 d_J}\cdot\left\{\frac{e^{\pm ik_0\beta_0^{(J)}x}}{\beta_o^{(J)}}-2i\sum_{m=1}^{\infty}\cos\frac{m\pi y}{d_J}\frac{e^{\mp k_0\gamma_m^{(J)}x}}{\gamma_m^{(J)}}\right\} \quad (7)$$

where

$$\gamma_m^{(J)}=\left[\left(\frac{m\pi}{k_0 d_J}\right)^2-\beta_0^{(J)2}\right]^{1/2}$$

$$\beta_0^{(J)}=\left(n_J^2-\alpha^2\right)^{1/2}. \quad (8)$$

For large m, $\gamma_m^{(J)}\approx m\pi/k_0 d_J$; thus, the sum tends toward the harmonic series as $(x,y)\to(0,0)$. This logarithmically singular behavior represents the dominant contribution in the slot for $|x|$ small and may be extracted by adding and subtracting the asymptotic form of the series in (7). Summing the dominant series and collecting results, we find

$$\phi^{(J)\pm}(x,y)=\frac{i\pi}{k_0 d_J}\left\{\frac{e^{\pm ik_0\beta_0^{(J)}x}}{\beta_0^{(J)}}-\frac{ik_0 d_J}{\pi}\cdot\left[\pm\frac{\pi x}{k_0 d_J}-\ln\left(2\cosh\frac{\pi x}{d_J}-2\cos\frac{\pi y}{d_J}\right)\right] -2i\sum_{m=1}^{\infty}\cos\frac{m\pi y}{d_J}\left[\frac{e^{\mp k_0\gamma_m^{(J)}x}}{\gamma_m^{(J)}}-\frac{e^{\mp m\pi x/d_J}}{m\pi/k_0 d_J}\right]\right\}. \quad (9)$$

The remaining series is convergent.

III. Approximate Narrow Slot Solution

The integral equations (6) are suitable for various numerical solutions or, because of the specific kernel involved, to an expansion of the unknown slot fields in terms of Chebyshev polynomials [25], [26]. However, if the slot is assumed to be small, that is, $(k_0 g)^2\ll 1$ and $(g/d_J)\ll 1$, then in the slot ($y=0$) where $|x|\le g$, the kernel is approximately given by

$$\phi^{(J)}(x-x')\approx -2\left\{\ln\frac{|x-x'|}{g}+\phi_0^{(J)}\right\}$$

$$\phi_0^{(J)}=\ln\frac{\pi g}{d_J}-\sum_{m=1}^{\infty}\left[\frac{\pi}{k_0 d_J\gamma_m^{(J)}}-\frac{1}{m}\right]-\frac{i\pi}{2k_0 d_J\beta_0^{(J)}}. \quad (10)$$

This "narrow slot" kernel allows the integral equations (6) to be solved directly, thus providing closed-form expressions for the desired field quantities (subject to the above restrictions).

The logarithmic kernel (10) is encountered in various small aperture problems (cf [25]), and experience indicates that a suitable gap field form is a $(g^2-x^2)^{-1/2}$ term times an expansion in powers of $(k_0 x)^n$. Physically, this accounts for the proper edge condition. Given the above narrow slot restrictions, it is necessary to keep only the first two terms ($n=0,1$) in $k_0 x$ to generate a consistent solution as well as preserve both even and odd slot field symmetries.

The integral equations (6) apparently contain three unknown field components: $E_x(x')$, $H_y(x')$, and $E_z(x')$. However, the three are related via

$$\partial_x E_z(x)=ik_0\alpha\left[E_x(x)-\frac{\eta_0}{\alpha}H_y(x)\right]. \quad (11)$$

Thus, only two independent components exist. Based upon these observations, we postulate the following solution:

$$E_x(x)=\frac{A_0+A_1k_0x}{(g^2-x^2)^{1/2}}$$

$$\frac{\eta_0}{\alpha}H_y(x)=\frac{A_0+B_1k_0x}{(g^2-x^2)^{1/2}}$$

$$E_z(x)=-ik_0^2\alpha(A_1-B_1)(g^2-x^2)^{1/2} \quad (12)$$

where (11) and the edge condition $E_z(\pm g)=0$ have been used to show that the leading term A_0 is the same for both $E_x(x)$ and $H_y(x)$ and that the integration constant implied by (11) is, in fact, zero.

Substituting (10) and (12) into our integral equations (6), and performing the integrations, yields a trio of equations for the unknown coefficients A_0, A_1, and b_1, namely,

$$2\pi(\epsilon_{r1}+\epsilon_{r2})A_1=\epsilon_{r2}E_{y,\mathrm{inc}}^{(2)}(x)-\epsilon_{r1}E_{y,\mathrm{inc}}^{(1)}(x)$$

$$4\pi\alpha B_1=-\eta_0H_{x,\mathrm{inc}}^{(2)}(x)+\eta_0H_{x,\mathrm{inc}}^{(1)}(x)$$

$$\pi\Delta A_0+\pi k_0x\left[(\epsilon_{r1}+\epsilon_{r2})A_1-2\alpha^2B_1\right]=\frac{\eta_0}{2i}\left(H_{z,\mathrm{inc}}^{(2)}(x)-H_{z,\mathrm{inc}}^{(1)}(x)\right) \quad (13)$$

where Δ is defined by

$$\Delta=\beta_0^{(1)^2}\left(\ln 2-\phi_0^{(1)}\right)+\beta_0^{(2)^2}\left(\ln 2-\phi_0^{(2)}\right). \quad (14)$$

Clearly, the incident fields need to be approximated consistent with the above discussion. The incident parallel-plate

TEM modes in the transverse plane are given by

$$E_{y,\mathrm{inc}}^{(J)\pm}(x) = E_J^{\pm} e^{\pm ik\beta_0^{(J)}x}$$
$$\boldsymbol{H}_{t,\mathrm{inc}}^{(J)\pm}(x) = \frac{1}{\eta_0}\left(\pm\beta_0^{(J)}\boldsymbol{a}_z - \alpha\boldsymbol{a}_x\right)E_J^{\pm}e^{\pm ik_0\beta_0^{(J)}x} \quad (15)$$

where $E_J^{\pm}$ denotes the magnitudes of the incident electric fields. From (13), it follows that only the leading term in the $(k_0x)^n$ expansion for $E_{y,\mathrm{inc}}^{(J)\pm}(x)$ and $H_{x,\mathrm{inc}}^{(J)\pm}(x)$ needs to be retained, whereas, for $H_{z,\mathrm{inc}}^{(J)\pm}(x)$, the first two terms are necessary. Thus

$$E_{y,\mathrm{inc}}^{(J)}(x) \approx E_J^+ + E_J^-$$
$$H_{c,\mathrm{inc}}^{(J)}(x) \approx \frac{-\alpha}{\eta_0}\left(E_J^+ + E_J^-\right)$$
$$H_{z,\mathrm{inc}}^{(J)}(x) \approx \frac{\beta_0^{(J)}}{\eta_0}\left(E_J^+ - E_J^-\right) + \frac{ik_0x\beta_0^{(J)^2}}{\eta_0}\left(E_J^+ - E_J^-\right). \quad (16)$$

Substituting (16) into (13) and solving for the coefficients A_0, A_1, and B_1, leads to the following approximate solution:

$$E_x(x) = \frac{i}{2\pi(g^2-x^2)^{1/2}} \cdot \left\{ \frac{\beta_0^{(1)}(E_1^+ - E_1^-) - \beta_0^{(2)}(E_2^+ - E_2^-)}{\Delta} + ik_0x\frac{\epsilon_{r1}(E_1^+ + E_1^-) - \epsilon_{r2}(E_2^+ + E_2^-)}{\epsilon_{r1}+\epsilon_{r2}} \right\}$$
$$H_y(x) = \frac{i\alpha}{2\pi\eta_0(g^2-x^2)^{1/2}} \cdot \left\{ \frac{\beta_0^{(1)}(E_1^+ - E_1^-) - \beta_0^{(2)}(E_2^+ - E_2^-)}{\Delta} + ik_0x\frac{(E_1^+ + E_1^-) - (E_2^+ + E_2^-)}{2} \right\}$$
$$E_z(x) = \frac{ik_0^2\alpha}{4\pi}\frac{(\epsilon_{r1}-\epsilon_{r2})}{(\epsilon_{r1}+\epsilon_{r2})}(g^2-x^2)^{1/2} \cdot \left\{ (E_1^+ + E_1^-) + (E_2^+ + E_2^-) \right\}. \quad (17)$$

A few observations are in order. The dominant logarithmic slot dependence, contained in Δ, was determined by A_0 (or $H_{z,\mathrm{inc}}$) in (13). Therefore, the primary coupling fields E_x and H_y are excited by the incident longitudinal magnetic field. In contrast, the longitudinal electric field is only weakly excited and, in the case of the equal dielectrics, disappears altogether. Thus, the scattered fields are essentially TE, or quasi-TE, which is not unexpected considering the TE nature of the assumed incident field distribution. Next, note that the distribution of the vertical electric field about the slot gives rise to two distinct symmetries. If $E_{y,\mathrm{inc}}^{(J)}(x)$ is indeed *symmetric* about the slot, i.e., $E_J^+ = E_J^-$, then the dominant terms drop out, and coupling is achieved via $E_{y,\mathrm{inc}}$ and $H_{x,\mathrm{inc}}$. If, however, an *asymmetric* distribution is the case, i.e., $E_J^+ \neq E_J^-$, then the dominant terms will be present and, as shall be seen in the next section, the fields excited by the secondary (symmetric) contributions may be neglected. Clearly, in the event $E_J^+ = -E_J^-$, the secondary terms drop out altogether. In the remaining discussion, these two cases, symmetric and asymmetric, will be examined separately with the emphasis on the dominant asymmetric case.

IV. The Scattered Fields

The slot fields (17) may now be substituted into (5) in order to evaluate the scattered fields. Specifically, we are interested in the parallel-plate TEM modes so it is sufficient to find $E_y^{(J)}(x,y)$. Defining a pair of convenient integrals allows $E_y^{(J)}(x,y)$ to be written as follows:

$$(\pm)E_y^{(J)\pm}(x,y) = \frac{1}{k_0}\left\{\partial_x T_1^{(J)\pm}(x,y) + ik_0\alpha T_2^{(J)\pm}(x,y)\right\} \quad (18)$$

where

$$T_1^{(J)\pm}(x,y) = \int_{-g}^{g}\phi^{(J)\pm}(x-x',y)E_x(x',0)\,dx'$$
$$T_2^{(J)\pm}(x,y) = \int_{-g}^{g}\phi^{(J)\pm}(x-x',y)E_z(x',0)\,dx'. \quad (19)$$

Recall that $(\pm)$ refers to the upper/lower plate region, whereas $\pm$ indicates the x-direction of propagation. In order to perform these integrations, the initial expression (7) for $\phi^{(J)\pm}(x,y)$ will be used. Allowing the observation point x to be outside the slot region introduces a strong exponential decay in the summand; therefore, convergence is no longer a problem as it was in the slot. Leaving $\boldsymbol{E}_t(x',0)$ in terms of the known coefficients A_0, A_1, B_1 yields, after integration

$$T_1^{(J)\pm}(x,y) = \frac{i\pi^2}{k_0d_J}\left\{\frac{e^{\pm ik_0\beta_0^{(J)}x}}{\beta_0^{(J)}}\left[A_0J_0(k_0\beta_0^{(J)}g) \mp ik_0gA_1J_1(k_0\beta_0^{(J)}g)\right] - 2i\sum_{m=1}^{\infty}\cos\frac{m\pi y}{d_J} \cdot \frac{e^{\mp ik_0\gamma_0^{(J)}x}}{\gamma_0^{(J)}}\left[A_0I_0(k_0\gamma_0^{(J)}g) \pm k_0gA_1I_1(k_0\gamma_m^{(J)}g)\right]\right\}$$

$$T_2^{(J)\pm}(x,y) = \frac{\alpha\pi^2}{k_0d_J}k_0g(A_1-B_1)\left\{\frac{e^{\pm ik_0\beta_0^{(J)}x}}{\beta_0^{(J)2}}J_1(k_0\beta_0^{(J)}g) - 2i\sum_{m=1}^{\infty}\cos\frac{m\pi y}{d_J}\frac{e^{\mp k_0\gamma_0^{(J)}x}}{\gamma_m^{(J)^2}}I_1(k_0\gamma_m^{(J)}g)\right\}. \quad (20)$$

It would be irrelevant to retain these full expressions in light of our previous narrow slot simplifications; thus, terms on the order of $(k_0g)^2$, with respect to unity, may be immediately discarded. This yields some simplification but an additional restriction is necessary to make the above expressions tractable. If the reflecting walls are "away" from the slot, then the evanescent higher order parallel-plate

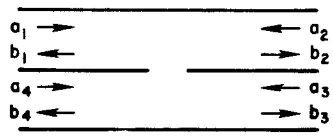


Fig. 3. Schematic of a four-port scattering junction.

modes will not contribute significantly due to reflection. Specifically, we will ignore the exponentially decaying terms, i.e.,

$$T_1^{(j)\pm}(x,y) \approx \frac{i\pi^2}{k_0\beta_0^{(j)}d_j}\left[A_0 J_0\left(k_0\beta_0^{(j)}g\right) \mp ik_0 g J_1\left(k_0\beta_0^{(j)}g\right)\right] e^{\pm ik_0\beta_0^{(j)}x}$$

$$T_2^{(j)\pm}(x,y) \approx \frac{\alpha\pi^2}{k_0 d_j}\frac{k_0 g}{\beta_0^{(j)2}}(A_1 - B_1) J_1\left(k_0\beta_0^{(j)}g\right) e^{\pm ik_0\beta_0^{(j)}x}. \tag{21}$$

For the dominant asymmetric case ($A_0 \neq 0$), neglecting terms like $(k_0 g)^2$ and taking the proper x-derivative in (18) gives

$$(\pm)E_y^{(j)\pm}(x,y) = \mp\frac{i\pi}{2k_0 d_j\Delta}\cdot\left[\beta_0^{(1)}(E_1^+ - E_1^-) - \beta_0^{(2)}(E_2^+ - E_2^-)\right] e^{\pm ik_0\beta_0^{(j)}x}. \tag{22}$$

In the symmetric case ($A_0 = 0$), the analogous result turns out to be

$$(\pm)E_y^{(j)\pm}(x,y) = \frac{i\pi^2(k_0 g)^2}{\gamma k_0\beta_0^{(j)}d_j(\epsilon_{r1}+\epsilon_{r2})}\left\{\left[2\epsilon_{rj}\epsilon_{r1} - \alpha^2(\epsilon_{r1}+\epsilon_{r2})\right](E_1^+ + E_1^-) + \left[2\epsilon_{rj}\epsilon_{r2} - \alpha^2(\epsilon_{r1}+\epsilon_{r2})\right]\cdot(E_2^+ + E_2^-)\right\} e^{\pm ik_0\beta_0^{(j)}x}. \tag{23}$$

The justification for neglecting the symmetric contribution when the asymmetric slot fields are excited is now apparent and amounts to the condition that

$$(k_0 g)^2 \ll \left[\ln\left(\frac{d_j}{g}\right)\right]^{-1}.$$

V. Scattering Matrices

A four-port network may be represented schematically as shown in Fig. 3. A scattering matrix S relates the input (A), and output (B) wave amplitudes via $B = SA$, where A and B are normalized such that input and output powers are given by AA^* and BB^*, respectively. The results of the previous sections may be used to characterize the slot coupling in terms of such a scattering matrix.

Begin with the incident fields (15) which determine A; the power carried in the transverse plane by the various TEM modes is

$$P_{\text{inc}}^{(j)\pm} = \pm\frac{\beta_0^{(j)}d_j}{2\eta_0}|E_j^\pm|^2. \tag{24}$$

Normalizing, according to

$$e_j^\pm = \left(\frac{\beta_0^{(j)}d_j}{2\eta_0}\right)^{1/2} E_j^\pm \tag{25}$$

so that the proper power relation holds implies that $a_1 = e_1^+$, $a_2 = e_1^-$, $a_3 = e_2^-$, and $a_4 = e_2^+$.

Consider next the scattered TEM mode for the asymmetric case; from (22) it follows that the transverse power is given by

$$P_s^{(j)\pm} = \pm\frac{\beta_0^{(j)}d_j}{2\eta_0}\left|\frac{\pi}{2k_0 d_j\Delta}\right|^2\cdot\left[\beta_0^{(1)}(E_1^+ - E_1^-) - \beta_0^{(2)}(E_2^+ - E_2^-)\right]^2. \tag{26}$$

Thus, analogous to (25), we may define a set of normalized scattered wave amplitudes

$$e_s^{(j)\pm} = \mp\frac{i\pi\beta_0^{(j)}}{2k_0 d_j\Delta}\left(\frac{d_j}{2\eta_0\beta_0^{(j)}}\right)^{1/2}\cdot\left[\beta_0^{(1)}(E_1^+ - E_1^-) - \beta_0^{(2)}(E_2^+ - E_2^-)\right] \tag{27}$$

where care has been taken to preserve the various polarities implied by (22). In terms of the normalized incident wave amplitudes $e_j^\pm$, the scattered wave amplitudes are given

$$e_s^{(j)\pm} = \pm\delta_j\left(\frac{d_j}{\beta_0^{(j)}}\right)^{1/2}\left[\left(\frac{\beta_0^{(1)}}{d_1}\right)^{1/2}(e_1^+ - e_1^-) - \left(\frac{\beta_0^{(2)}}{d_2}\right)^{1/2}(e_2^+ - e_2^-)\right] \tag{28}$$

where

$$\delta_j = \frac{-i\pi\beta_0^{(j)}}{2k_0 d_j\Delta}. \tag{29}$$

This implies that $b_1 = e_s^{1+}$, $b_2 = e_s^{1-}$, etc., and it follows from the definition of the a_i that the asymmetric scattering matrix $S^{(0)}$ is

$$S^{(0)} = \begin{pmatrix} -\delta_1 & 1+\delta_1 & -\epsilon & \epsilon \\ 1+\delta_1 & -\delta_1 & \epsilon & -\epsilon \\ -\epsilon & \epsilon & -\delta_2 & 1+\delta_2 \\ \epsilon & -\epsilon & 1+\delta_2 & -\delta_2 \end{pmatrix} \tag{30}$$

where $\epsilon = (\delta_1\delta_2)^{1/2}$. Because (28) only accounts for the scattered waves excited by the slot, the unscattered incident fields must also be included, which accounts for the $1+\delta_j$ off-diagonal terms. In a precisely analogous fashion, the symmetric case scattering matrix $S^{(e)}$ is found to be

$$S^{(e)} = \begin{pmatrix} \rho_1 & 1+\rho_1 & -\sigma & -\sigma \\ 1+\rho_1 & \rho_1 & -\sigma & -\sigma \\ -\sigma & -\sigma & \rho_2 & 1+\rho_2 \\ -\sigma & -\sigma & 1+\rho_2 & \rho_2 \end{pmatrix} \tag{31}$$

where the following auxiliary quantities have been defined:

$$\rho_j = \zeta_j\left[2\epsilon_{rj}^2 - \alpha^2(\epsilon_{r1}+\epsilon_{r2})\right]$$

$$\sigma = (\zeta_1\zeta_2)^{1/2}\left[2\epsilon_{r1}\epsilon_{r2} - \alpha^2(\epsilon_{r1}+\epsilon_{r2})\right]$$

$$\zeta_j = \frac{-i\pi(k_0 g)^2}{8k_0 d_j\beta_0^{(j)}(\epsilon_{r1}+\epsilon_{r2})}. \tag{32}$$

Notice that both matrices are symmetric as expected for a reciprocal network, and that the cross-coupling coefficients ϵ_1 and σ feature the individual guide parameters in a balanced fashion. The matrices are unitary [27] which reflects a lossless junction.

VI. The Modal Equations

The modal equations for two waveguiding structures coupled by a narrow longitudinal slot may now be considered. In order to model a particular waveguide, a reflection matrix Γ must be specified which characterizes the boundaries to the parallel-plate sections. The requirement of constructive interference leads to a transverse resonance condition, namely,

$$\det(I - \Gamma S) = 0. \tag{33}$$

Specifically, if we let the reflections undergone by backward and forward waves be denoted Γ_j^+ and Γ_j^-, respectively, then Γ is a simple diagonal matrix.

In the dominant slot coupled case, with $S^{(0)}$ given by (30), the transverse resonance condition (33) implies that

$$\det\begin{vmatrix} 1+\delta_1\Gamma_1^+ & -(1+\delta_1)\Gamma_1^+ & \epsilon\Gamma_1^+ & -\epsilon\Gamma_1^+ \\ -(1+\delta_1)\Gamma_1^- & 1+\delta_1\Gamma_1^- & -\epsilon\Gamma_1^- & \epsilon\Gamma_1^- \\ \epsilon\Gamma_2^- & -\epsilon\Gamma_2^- & 1+\delta_2\Gamma_2^- & -(1+\delta_2)\Gamma_2^- \\ -\epsilon\Gamma_2^+ & \epsilon\Gamma_2^+ & -(1+\delta_2)\Gamma_2^+ & 1+\delta_2\Gamma_2^+ \end{vmatrix} = 0. \tag{34}$$

Solving yields a modal equation which may be formally written as

$$F_1F_2 - \delta_1\delta_2G_1G_2\Gamma_1^-\Gamma_2^- = 0 \tag{35}$$

where

$$F_j = (1+\delta_j\Gamma_j^-)G_j - (1-\Gamma_j^-)^2\Gamma_j^+$$
$$G_j = \Gamma_j^+ + \Gamma_j^- - 2\Gamma_j^+\Gamma_j^- \tag{36}$$

Although this is the desired result for the asymmetric case, the dependence on α is less than clear. Therefore, some further simplification is beneficial. Referring to Fig. 4, the reflections may be represented in an exponential form

$$\Gamma_j^{\pm} = e^{\iota(\phi_j \pm \psi_j)} \tag{37}$$

where $\phi_j = 2k_0\beta_0^{(j)}l_j - \chi_j(\alpha)$ accounts for propagation to and from the slot center plus any phase change $\chi_j(\alpha)$ at the reflecting boundary, and $\psi_j = 2k_0\beta_0^{(j)}\,\mathrm{off}_j$ compensates for an off-center slot. After some manipulation, the substitution of (37) into the modal equation (35) yields

$$\frac{\pi}{2}\left\{\frac{\beta_0^{(1)}}{k_0d_1}\sin\phi_2(\cos\phi_1 - \cos\psi_1) + \frac{\beta_0^{(2)}}{k_0d_2}\sin\phi_1(\cos\phi_2 - \cos\psi_2)\right\}$$
$$- \sin\phi_1\sin\phi_2\sum_{j=1}^{2}\beta_0^{(j)^2}\left(\ln\frac{2d_j}{\pi g} + \sum_{m=1}^{\infty}\left[\frac{\pi}{k_0\gamma_m^{(j)}d_j} - \frac{1}{m}\right]\right) = 0. \tag{38}$$

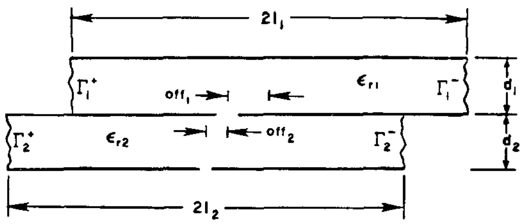


Fig. 4. Slot scattering cross section.

For the special case of equal ($\epsilon_{r1} = \epsilon_{r2} = \epsilon$, $d_1 = d_2 = d$, etc.) guides about a centered slot ($\psi_1 = \psi_2 = 0$), the modal equation reduces to

$$\beta_0\sin^2\phi\left\{\frac{\pi}{k_0d}\tan\frac{\phi}{2} + 2\beta_0\left(\ln\frac{2d}{\pi g} + \sum_{m=1}^{\infty}\left[\frac{\pi}{k_0\gamma_m d} - \frac{1}{m}\right]\right)\right\} = 0. \tag{39}$$

The symmetric case may be similarly analyzed. Because the vertical electric field is required to be laterally symmetric about the slot, we have by default that $\Gamma_j^+ = \Gamma_j^- = \Gamma_j$ and that $\psi_j = 0$. One finds that

$$(1+\Gamma_1)(1+\Gamma_2)\left[(1-\Gamma_1)(1-\Gamma_2) - 2\rho_1\Gamma_1(1-\Gamma_2) - 2\rho_2\Gamma_2(1-\Gamma_1)\right] = 0. \tag{40}$$

No form equivalent to (38) is readily available, although one may result if the proper (negligible) terms of order $(k_0g)^4$ are added. For identical guides ($\Gamma_1 = \Gamma_2 = \Gamma$, etc.), the symmetric case modal equation reduces to

$$\sin^2\phi\left[1 - \frac{4\rho\Gamma}{1-\Gamma}\right] = 0. \tag{41}$$

A few special cases may now be considered. In the case of no slot ($g = 0$ or no coupling), both cases reduce to

$$\sin\phi_1\sin\phi_2 = 0 \tag{42}$$

where the slot location is now irrelevant ($\psi_1 = \psi_2 = 0$). The roots to $\sin\phi_j = 0$ generate the unperturbed TE_{mo} modes in the respective guides. Familiarity with these simple solutions often helps to identify the basic nature of a slot perturbed mode.

As $g \to 0$, the asymmetric modal equation is asymptotic to

$$\sin\phi_1\sin\phi_2\left\{\beta_0^{(1)^2}\ln\frac{2d_1}{\pi g} + \beta_0^{(2)^2}\ln\frac{2d_2}{\pi g}\right\} = 0. \tag{43}$$

Setting the term in parenthesis to zero yields a solution for α which tends toward

$$\alpha \sim \left(\frac{n_1^2 + n_2^2}{2}\right)^{1/2}. \tag{44}$$

This is recognized as the zeroth-order solution for propagation along a slotline (cf [28]), as well as the familiar result due to Coleman [29] for the dual problem of a narrow conducting strip between two dielectric media. In essence, as the slot width decreases, the fields become tightly bound to the slot and are no longer affected by the outer conductors, thus the slot line behavior.

VII. Conclusion

The mutual excitation between a pair of parallel-plate waveguides has been analyzed taking advantage of the restriction that the slot (the coupling mechanism) be narrow. The primary results are modal equations valid for a variety of planar waveguide structures. This building block approach is both simple and flexible. The second part of this paper applies the above canonical problem to some novel coupling schemes and waveguide structures.

References

[1] H. Lamb, *Hydrodynamics*, 6th ed. New York: Dover Press, 1945, pp. 492–520.

[2] Lord Rayleigh, "On the incidence of aerial and electric waves upon small obstacles in the form of ellipsoids or elliptic cylinders, and on the passage of electric waves through a circular aperture in a conducting screen," in *Scientific Papers*, vol. IV. Cambridge: Cambridge University Press, 1903, pp. 306–326.

[3] Lord Rayleigh, "On the passage of waves through apertures in plane screens and allied problems," in *Scientific Papers*, vol. IV. Cambridge: Cambridge University Press, 1903, pp. 283–296.

[4] H. A. Bethe, "Theory of diffraction by small holes," *Phys. Rev.*, vol. 66, pp. 163–182, Oct. 1944.

[5] C. L. Pekeris, "Comments on Bethe's theory of diffraction of electromagnetic waves by small holes," *Phys. Rev.*, vol. 66, p. 351, Dec. 1944.

[6] A. F. Stevenson, "Theory of slots in rectangular wave-guides," *J. Appl. Phys.*, vol. 19, pp. 24–38, Jan. 1948.

[7] A. A. Oliner, "The impedance properties of narrow radiating slots in the broad face of rectangular waveguide," *IRE Trans. Antennas Propagat.*, vol. AP-5, pp. 4–20, Jan. 1957.

[8] L. Lewin, "Some observations on waveguide coupling through medium-sized slots," *Proc. Inst. Elec. Eng.*, vol. 107C, pp. 171–178, Feb. 1960.

[9] A. A. Oliner, "Equivalent circuits for small symmetrical longitudinal apertures and obstacles," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 72–80, Jan. 1960.

[10] A. J. Sangster, "Variational method for the analysis of waveguide coupling," *Proc. Inst. Elec. Eng.*, vol. 112, pp. 2171–2179, Dec. 1965.

[11] R. Levy, "Directional couplers," in *Advances in Microwaves*, vol. 1, L. Young, Ed. New York: Academic Press, 1966, pp. 115–209.

[12] B. N. Das and G. S. Sanyal, "Network parameters of a waveguide broad wall-slot radiator," *Proc. Inst. Elec. Eng.*, vol. 117, pp. 41–44, Jan. 1970.

[13] V. M. Pandharipande and B. N. Das, "Equivalent circuit of a narrow-wall waveguide slot coupler," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 800–804, Sept. 1979.

[14] S. E. Miller, "Coupled wave theory and waveguide applications," *Bell Syst. Tech. J.*, vol. 33, pp. 661–719, 1954.

[15] S. Ramo, J. R. Whinnery, and T. Van Dozer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1965.

[16] D. Marcuse, *Theory of Dielectric Optical Waveguides*. New York: Academic Press, 1974.

[17] D. C. Chang and E. F. Kuester, "Total and partial reflection from the end of a parallel-plate waveguide with extended dielectric slab," *Radio Sci.*, vol. 16, pp. 1–13, 1981.

[18] D. C. Chang, "Analytical theory of an unloaded rectangular microstrip patch," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 52–64, 1981.

[19] E. F. Kuester, R. T. Johnk, and D. C. Chang, "The thin-substrate approximation for reflection from the end of a slab-loaded parallel-plate waveguide with application to microstrip patch antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 910–917, Sept. 1982.

[20] S. T. Peng and A. A. Oliner, "Guidance and leakage properties of a class of open dielectric waveguides: Part I—Mathematical formulations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 843–855, Sept. 1981.

[21] A. A. Oliner, S. T. Peng, T. I. Hsu, and A. Sanchez, "Guidance and leakage properties of a class of open dielectric waveguides: Part II—New physical effects," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 855–869, Sept. 1981.

[22] C. J. Bouwkamp, "Diffraction theory," *Rep. Progr. Phys.*, vol. 17, pp. 75–100, 1954.

[23] J. Van Bladel, "Small hole coupling of resonant cavities and waveguides," *Proc. Inst. Elec. Eng.*, vol. 117, pp. 1098–1104, 1970.

[24] D. C. Chang, "A general theory of small radiating apertures in the outer sheath of a coaxial cable," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 674–683, Sept. 1968.

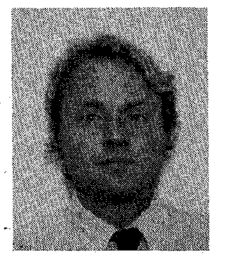
[25] F. G. Tricomi, *Integral Equations*. New York: Interscience, 1957.

[26] J. C. Tippet and D. C. Chang, "Characteristic impedance of a rectangular coaxial line with offset inner conductors," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 876–883, Nov. 1978.

[27] P. F. Wilson, "On slot-coupled waveguide and transmission line structures," Ph.D. thesis, Univ. of Colorado, Boulder, CO, May 1983.

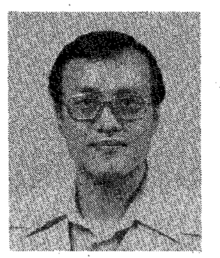
[28] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.

[29] B. L. Coleman, "Propagation of electromagnetic disturbances along a thin wire in a horizontally stratified medium," *Phil. Mag.*, vol. 41, ser. 7, pp. 276–288, Mar. 1950.



Perry F. Wilson (S'78–M'83) was born in Palo Alto, CA. He received the B.S. degree in mathematics from Stanford University in 1974, the M.S. degree in applied mathematics, and the M.S. and Ph.D. degrees in electrical engineering from the University of Colorado, Boulder, CO, in 1977, 1978, and 1983, respectively. He held a NRC Postdoctoral Research Associateship with the Electromagnetic Fields Division, National Bureau of Standards (NBS) in Boulder, CO, from July 1983 to January 1985.

Since February 1985, he has been a staff member with NBS. His technical interests include waveguides, remote sensing, and EMC/EMI related problems.



David C. Chang (M'67–SM'76–F'85) was born in Hupeh, China, on September 9, 1941. He received the B.S. degree in electrical engineering from Cheng Kung University, Tainan, Taiwan, China, in 1961, and the M.S. and Ph.D. degrees in applied physics from Harvard University, Cambridge, MA, in 1963 and 1967, respectively.

He joined the University of Colorado, Boulder, in 1967, and is now a Professor of Electrical Engineering and, since 1982, Chairman of the Electrical and Computer Engineering Department. In 1972, he was a Visiting Professor at Queen Mary College, University of London, London, England.

Dr. Chang is a member of Commissions A, B, C, and E of the International Union of Radio Science, and member-at-large of its U.S. National Committee. He was an Associate Editor of IEEE Transactions on Antennas and Propagation, the Coordinator of the IEEE/APS Distinguished Lecturer's Program, Chairman of the Subcommittee on Microwave Field Theory (1975–1978) of IEEE Professional Society on Microwave Theory and Techniques. He is a consultant to Kaman Science Corporation, Colorado Springs, CO, and to Teledyne Micronetics, San Diego, CA.